Q.P. Code: 19HS0830 **R19**

Reg. No: | | | | | | | | | | |

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR

(AUTONOMOUS)

B.Tech I Year I Semester Supplementary Examinations July-2022

ALGEBRA AND CALCULUS

(Common to All)

Time: 3 hours Max. Marks: 60

(Answer all Five Units **5 x 12 = 60** Marks)

UNIT-I

1	**a**	For what values of k the equations $x+y+z=1,\ 2x+y+4z=k,$ $4x+y+10z=k^2$ have a solution and then solve.	**L1**	**6M**
	b	Verify the Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}8 & -8 & 2\\ 4 & -3 & -2\\ 3 & -4 & 1\end{bmatrix}$.	**L2**	**6M**

OR

2	**a**	Find the Eigen values of the matrix $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$. Hence find the Eigen values of A^2-2A+I.	**L3**	**5M**
	b	Diagonalise the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ -4 & 4 & 3\end{bmatrix}$.	**L2**	**7M**

UNIT-II

3	**a**	Show that for any $x>0, 1+x<e^x<1+xe^x$, using Lagrange's mean value theorem.	**L3**	**6M**
	b	Expand $\sin x$ in powers of $(x-\pi/2)$ up to term containing $\left(x-\frac{\pi}{2}\right)^4$ using Taylor's series.	**L1**	**6M**

OR

4	**a**	Verify Rolle's theorem for the function $f(x)=x(x+3)e^{-\frac{x}{2}}$, *in* $[-3, 0]$.	**L3**	**6M**
	b	Calculate the approximate value of $\sqrt{10}$ using Taylor's series, correct to 4 decimal places.	**L1**	**6M**

UNIT-III

5	**a**	If $U=\log(x^3+y^3+z^3-3xyz)$ prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 U=\frac{-9}{(x+y+z)^2}$.	**L1**	**6M**
	b	If $u=x\sqrt{1-y^2}+y\sqrt{1-x^2}$ and $v=\sin^{-1}x+\sin^{-1}y$ then show that u and v are functionally dependent.	**L2**	**6M**

OR

6	**a**	Find the stationary points of $u(x,y)=\sin x.\sin y.\sin(x+y)$, where $0<x<\pi$, $0<y<\pi$ and find the maximum of u.	**L2**	**6M**
	b	If $u=x+3y^2-z^3, v=4x^2yz, w=2z^2-xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0).	**L1**	**6M**

UNIT-IV

7	**a**	Change the order of integration and evaluate $\int_0^1\int_{x^2}^{2-x} xydydx$.	**L3**	**6M**
	b	Evaluate the integral $\int_0^2\int_0^{\sqrt{2x-x^2}}(x^2+y^2)dydx$ by transforming into polar coordinates.	**L2**	**6M**

OR

8	**a**	Evaluate the integral $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}}\frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$.	**L1**	**6M**
	b	Evaluate the integral by changing the order of integration $\int_0^a\int_{\frac{x}{a}}^{\sqrt{\frac{x}{a}}}(x^2+y^2)dydx$.	**L2**	**6M**

UNIT-V

9	**a**	Express the integral $\int_0^1\frac{x}{\sqrt{1-x^2}}dx$ in terms of beta function.	**L3**	**6M**
	b	Show that $\int_0^\infty x^4e^{-x^2}dx=\frac{3\sqrt{\pi}}{8}$.	**L2**	**6M**

OR

10	**a**	Show that $\int_0^\infty e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$.	**L2**	**6M**
	b	Prove that $\int_0^1\sqrt{1-y^4}dy=\frac{1}{2}\beta\left(\frac{1}{4},\frac{3}{2}\right)$.	**L3**	**6M**

*** END ***